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Modeling of light coupling effect using tunneling theory based on particle properties of light

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Abstract Based on the wave-particle duality of light, the Schrödinger equation for a photon as a particle is established to treat the light coupling effect by introducing concepts of the virtual mass and potential for a photon, which is different from the previous method that uses the analogy with quantum mechanics. The virtual mass is physically correlated to the kinetic energy according to Einstein's energy–momentum relation. As a consequence, this new model has merits of physical simplicity and analytic nature. This new model can well explain the exponential dependence of the light coupling effect on the physical parameters in coupled waveguides on, which can be observed in the experimental and simulation data reported in the literature. Moreover, an explicit expression for the coupling length (coefficient) on the effects of physical parameters can be obtained by virtue of this new model, whereas the previous modal approach and the coupled-wave model resulted in implicit expressions. This new model does not only give a better physical understanding but also offers a possibility to design and fabricate optic devices based on the light coupling effect by optimizing the physical parameters.

Keywords Coupling · Tunneling · Waveguide · Wave-particle duality

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1 Introduction

Coupling effect in waveguides (e.g., the evanescent coupling between two parallel optical waveguides), which enables the transfer of light from one waveguide to an adjacent waveguide, is very important for a variety of applications such as switching, power division, dispersion compensation, and frequency (polarization) selection (Zheng et al. 2015; Moghaddam et al. 2010; Kuchinsky et al. 2002; Li et al. 2010; Wang et al. 2010; Kumar et al. 1985; Mortensen et al. 2003; Uthayakumar et al. 2012; Obayya et al. 2000; Singh 1999; Gorajoobi et al. 2013; Chaudhari and Gautam 2000; Headley et al. 2004; Suzuki et al. 1994; Song et al. 2011; Sha et al. 2015; Henry and Verbeek 1989; Imotijevic et al. 2007; Yu et al. 2014; Xu et al. 2014; Marom et al. 1984; Djordjev et al. 2002; Zhou et al. 2013; Ali et al. 2008; Bird 1990; Shakouri et al. 1998; Sun 2007; Noda et al. 1981). How the coupling length changes with the width of the separation layer on the coupling structure was studied in Zheng et al. (2015), Li et al. (2010), Singh (1999), Gorajoobi et al. (2013), Chaudhari and Gautam (2000), Suzuki et al. (1994), Song et al. (2011), Sha et al. (2015), Imotijevic et al. (2007), Marom et al. (1984), Yariv (1973). The relation between the coupling length and the wavelength was reported in Moghaddam et al. (2010), Kumar et al. (1985), Mortensen et al. (2003), Uthayakumar et al. (2012), Gorajoobi et al. (2013), Henry and Verbeek (1989), whereas the relation between the coupling length and the relative dielectric constant of the separation layer was reported in Wang et al. (2010), Obayya et al. (2000), Sun (2007), Noda et al. (1981), and the relation between the coupling length and the width of the waveguide was reported in Yu et al. (2014), Xu et al. (2014). Moreover, the relation between the coupling coefficient and the width of the separation layer was studied in Li et al. (2010), Djordjev et al. (2002), Zhou et al. (2013), Ali et al. (2008), Bird (1990), while the relation between the coupling coefficient and the frequency (wavelength) of light was investigated in Moghaddam et al. (2010), Shakouri et al. (1998).

Control of the coupling length is certainly one deciding factor to the success of optic devices based on directional couplers. Note that the method to obtain the coupling length is approximate because there is not an exact analytical solution to the wave equation (Kumar et al. 1985). The modal approach (Yariv 1973) and the coupled-wave approach (Suematsu and Kishino 1977) are two approaches to the problem of two coupled waveguides. In addition, an approach using the eigenvalue caused by the linear defect in the photonic crystal was proposed to determine the coupling length (Kuchinsky et al. 2002). One drawback of these approaches is that the physical origin of the coupling effect is unclear and there is not exact analytical solution, although their results are consistent with the experiments and simulations. A better understanding on the coupling effect can lead to optical directional couplers, which are important building blocks in photonic circuits, with better performances but simpler fabrication processes.

In studies of photonic circuits and optical devices, most researchers treat the optical problem or the electromagnetic problem using a close mathematical analogy between the Schrödinger equation and the Helmholtz equation for electromagnetic wave (Joannopoulos et al. 2011; Olkhovsky 2014; Keller 2001; Lenstra and van Haeringen 1990; Dragoman and Dragoman 2004 and the references therein). Over several decades of work, researchers have only recently begun to understand the true nature of such a wave function for a photon (Smith and Raymer 2007 and the references therein). There is a little study on photonic circuits and optical devices via using the concept of the particle properties of light. In this work, the particle properties of light are adopted to establish the Schrödinger equation for a photon. Note that the particle tunneling can be also treated as an evanescent wave coupling effect that occurs in quantum mechanics, while an evanescent wave coupling in optics refers to the coupling between two waveguides. Both the particle tunneling theory and the uncertainty principle are utilized to explain the light coupling effect in this paper.

The wave-particle duality holds that matter and light exhibit properties of both waves and particles. The purpose of this work is to shed some light on the coupling effect by virtue of the tunneling theory and the uncertainty principle in quantum mechanics based on the particle properties of light, which is different from the previous method that uses a close mathematical analogy between the Schrödinger equation and the Helmholtz equation to treat the optic problem or the electromagnetic problems. In particular, a new physical model relevant to the coupling effect in coupled waveguides according to the uncertainty principle and the tunneling theory is proposed. Through its physical simplicity and analytic nature, such a new model can be applied to describe the experimental results and provide the knowledge on how the physical parameters of coupled waveguides can affect the performances.

This new model illustrates that the physical origin of the light coupling effect can be explainable by the particle tunneling theory and the uncertainty principle because light is also a particle. In addition, a concept of potential can be developed to measure the effective dielectric constant for materials using the coupling effect of light. This new model also states that the coupling length and the coupling efficient can be modulated or tuned by choosing or modulating the width of waveguide, the separation between the waveguides, and the effective dielectric constant of the separation. Hence, optical directional couplers can be reasonably designed to achieve desirable performance by virtue of this new model. One may find that this new model is not only physical simplicity but also has great value in practical applications.

2 Theory

Electromagnetic field or light in a source-free dielectric space is described by Maxwell's equations (e.g., Fitzpatrick 2008)

$$\nabla \cdot \vec{D} = \rho = 0 \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\nabla \times \vec{B} = \mu\varepsilon \frac{\partial\left(\vec{E}\right)}{\partial t} = \frac{1}{c^2}\frac{\partial\left(\vec{E}\right)}{\partial t} \tag{4}$$

where ρ is the free charge density, μ is the permittivity of the dielectric space, ε is the permeability of the dielectric space, c is the light velocity in the dielectric space, t is the time, $\vec{D}$ is the displacement field vector, $\vec{B}$ is the magnetic field vector, and $\vec{E}$ is the electric field vector. Noting that $\nabla \times \nabla \times \vec{E} = -\left(1/c^2\right)\left(\partial^2\vec{E}\big/\partial t^2\right)$ and $\nabla \times \nabla \times \vec{E} = \nabla\left(\nabla \cdot \vec{E}\right) - \nabla^2\vec{E}$, we get

$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \tag{5}$$

For a plane wave, $\vec{E}(x,t) = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$, propagating in the x-direction in a lossless medium space, Eq. (5) becomes

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \left(k^2 - \frac{\omega^2}{c^2}\right)\vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)} = 0 \tag{6}$$

where $\vec{k}$ is the wave vector, and ω is the angular frequency. Equation (6) requires $\left(k^2 - \omega^2/c^2\right) = 0$. Considering the photon energy of $E = hv = \hbar\omega$ and its momentum of $p = h/\lambda = \hbar k$ (herein h is the Planck constant, $\hbar$ is the reduced Planck constant, and λ is the wavelength of light), Eq. (6) can be rewritten as

$$\left(p^2 - \frac{E^2}{c^2}\right)\vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)} = 0 \tag{7}$$

The Schrödinger equation is used to describe a physical system with quantum effects (such as wave-particle duality) being significant. There are two key parameters in the Schrödinger equation. One is the effective mass of the particle, the other is the potential. Thus, a virtual mass and the potential for a photon is need to be introduced for building a Schrödinger equation of a photon. Such a virtual mass and the potential can be introduced by comparing the Maxwell's equations for a photon and the assumption that the potential is zero when a photon travels in vacuum. Note that the photon momentum is determined by the wavelength, it is correlated to the medium where a photon travels. Usually, the mass does not change with the potential. Therefore, a virtual mass of a photon is determined by the frequency of the light and independent of the medium where a photo travels. On the other hand, the potential should be directly dependent of the medium where a photo travels. When a virtual mass and the potential for a photon have been given, a Schrödinger equation of a photon can be built, And thus we can totally copy the concept of tunneling and the methods how to determine the tunneling in quantum mechanics to study the coupling effect in waveguides via using the tunneling of a photon. Detail discuss will be found in the following.

2.1 A Schrödinger equation for a photon

According to the wave-particle duality of light, the light is both a particle and a wave. Thus the coupling effect in waveguides is similar to the case of a particle tunneling in double well potential when the light is treated as a particle. Then we can study the photon tunneling in the light of the methods used in the literature on tunneling in double well potential (e.g., Song 2008; Verguilla-Berdecia 1993; Rastelli 2012; Khuat-duy and Leboeuf 1993; Weiss et al. 1987; Benderskii and Kats 2002; Gangopadhyaya et al. 1993; Park et al. 1998; Jelic and Marsiglio 2012; Kierig et al. 2008).

Assume that a photon can be treated as a non-relativistic particle moving along the x-direction, and a particle in the free space can be described as a plane wave $\psi(x,t) = Ae^{\frac{i}{\hbar}(px-Et)}$ with $E = \frac{p^2}{2m} + V$.where m is the effective mass of a particle, and V is the potential of the particle (e.g., Shankar 2012). Note that such an effective mass is a virtual mass not a realistic mass for a photon. In order to build a wave equation based on the particle properties of the wave-particle duality of light, such a virtual mass (m) for a photon is introduced into the Schrödinger equation similar to the effective electron mass, which is used to characterize the kinetic energy of a photon when it is treated as a particle (In the following, we will discuss it in detail). Therefore, one obtains (e.g., Shankar 2012)

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right)\psi(x,t) \tag{8}$$

Because such a "particle" is a photon, it is thus also a wave. At the same time, one can also obtain

$$pc = E = \frac{p^2}{2m} + V \tag{9}$$

2.2 Potential for a photon

When the photon momentum $p = h/\lambda = \hbar k$ is the largest, i.e., the particle moves the fastest, the corresponding wavelength is the shortest and the potential is the lowest (in the valleys). It is well-known that the photon moves the fastest in the vacuum. Therefore, the potential in the vacuum for a photon is defined as zero. According to Eq. (9), one obtains

$$m = \frac{p}{2c_0} = \frac{\hbar\omega}{2(c_0)^2} \tag{10}$$

where c_0 is the light velocity in the vacuum. According to Einstein's energy–momentum relation, $E_{tot} = E_K^2 + m_0c^2$(Here E_{tot} is total energy of a particle, E_{K} is the sum of the kinetic energy and the potential energy, m_0 is the rest mass of a photon). Note that the rest mass of a photon is zero, thus $pc = E_{tot} = \frac{1}{2}mc^2$.we can also obtain the same expression as Eq. 10. Note that $E_{tot} = (pc)^2 + m_0c^2 = E_K^2 + m_0c^2$, the Lorentz invariance is $-m_0c^2$ the if the system is static for a photon (which means that the momentum is zero).

When a photon is moving in a dielectric space, the velocity is less than the light velocity in the vacuum, and thus it can be assumed that such a decrease in the velocity is caused by the potential. Therefore, according to Eq. (9), the potential for a photon in a dielectric material can be obtained as

$$V = pc - \frac{p^2}{2m} = \hbar\omega\left(1 - \frac{\mu\varepsilon}{\mu_0\varepsilon_0}\right) \tag{11}$$

where μ_0 and ε_0 are the permittivity and permeability of the vacuum, respectively.

Considering that the potential in the vacuum is the lowest and defined as zero, a photon behaves like a hole when $\varepsilon\mu \geq \varepsilon_0\mu_0$, and an electron when $\varepsilon\mu \leq \varepsilon_0\mu_0$. Hence, when $\varepsilon\mu \geq \varepsilon_0\mu_0$, the barrier height ϕ seen by a photon moving in a dielectric space is given as

$$\varphi = \hbar\omega \frac{\mu\varepsilon}{\mu_0\varepsilon_0} \tag{12}$$

2.3 Hole-like carrier tunneling

A photon moving through a dielectric material can be treated as a hole-like carrier tunneling. The time-independent one-dimensional Schrödinger equation for a photon moving through a dielectric material with $\varepsilon\mu \geq \varepsilon_0\mu_0$ can be written as (e.g., Shankar 2012)

$$\left[\nabla^2 + \frac{2m}{\hbar^2}E\right]\psi(x) = 0 \quad x < 0 \text{ or } x > b \tag{13}$$

$$\left[\nabla^2 + \frac{2m}{\hbar^2}(E - \varphi)\right]\psi(x) = 0 \quad 0 \leq x \leq b \tag{14}$$

where b denotes the thickness of the dielectric material. Using the usual plane wave solution method, the wave functions in three regions can be found in Shankar (2012) and the "Appendix". Therefore, the tunneling coefficient is given as

$$T_C = \frac{(2kk_B)^2}{(2kk_B)^2 + (k^2 + k_B^2)^2 \sinh^2(k_B b)} \tag{15}$$

If $k_B b \gg 1$ (such a condition can be satisfied when $b \geq \lambda$ or $b \approx \lambda$), $\sinh(k_B b) \to e^{k_B b}/2$ and Eq. (19) becomes

$$T_C \to \frac{(2kk_B)^2}{(k^2 + k_B^2)^2 (e^{k_B b}/2)^2} \to \left(\frac{4kk_B}{k^2 + k_B^2}\right)^2 e^{-2k_B b} \to \frac{1}{\omega^2 \mu\varepsilon} e^{-2\omega\sqrt{\mu\varepsilon} b} \tag{16}$$

2.4 Photon tunneling time

The tunneling time is the time that a particle takes to transverse a spatial region. The concept of the tunneling time in related research has remained controversial over the past 80 years (Stuchebrukhov 2016; Yang and Guo 2016; Kullie 2016; Winful 2006; Büttiker and Landauer 1982; Steinberg 1995 and the references therein). Previous studies have demonstrated that the physics of tunneling is essentially identical for classical light waves and quantum mechanical waves (Steinberg 1995 and the references therein). In this study, the transmission time can be described as Steinberg (1995)

$$\tau_T = \frac{m}{\hbar k}\frac{1}{T}\int_0^b dx\left(Ae^{k_B x} + Be^{-k_B x}\right)\left(Ae^{-k_B x} + Be^{k_B x}\right) \tag{17}$$

According to the above equation and $|A/B| = e^{2k_B b}$, it can be concluded that the main contribution to the tunneling time is the A^2 term. Therefore, the above equation can be approximated as

$$
\begin{aligned}
\tau_T &\approx \frac{mb}{\hbar k}\left|\frac{ik(k_B-ik)^2e^{2k_Bb}}{k_B\left[(k_B+ik)^2e^{-k_Bb}-(k_B-ik)^2e^{k_Bb}\right]}\right| \\
&= \frac{mb}{\hbar k}\left|\frac{k(k_B-ik)^2e^{k_Bb}}{k_B\left[(k_B+ik)^2e^{-2k_Bb}-(k_B-ik)^2\right]}\right|
\end{aligned} \tag{18}
$$

If $k_Bb > > 1$ (e.g., $b \geq \lambda$ or $b \approx \lambda$), one can has: $e^{-2k_Bb} = 0$. Therefore, Eq. (22) can be further approximated as

$$
\tau_T = \frac{mb}{\hbar k_B}e^{k_Bb} \to e^{k_Bb} \tag{19}
$$

2.5 Photon tunneling in double waveguides

In double waveguides with a separation layer, the wave functions in five regions can be found in Shankar (2012) and the "Appendix". Matching the wave functions and their derivatives at the boundaries, one can obtain the tunneling coefficient:

$$
T_C = \left|\frac{4k_B^2}{\left(e^{2ikw_1}-e^{ikb}\right)\left[(ik-k_B)^2 - e^{2ikw_2}(ik+k_B)^2\right]}\right|e^{-2k_B(2w_1+b)} \to e^{-2\omega\sqrt{\mu\varepsilon}(2w_1+b)} \tag{20}
$$

where w_1, w_2, and b are the width of waveguide 1, the width of waveguide 2, and the thickness of the separation layer, respectively.

Noting that the power transfer ratio between the two waveguides is defined as the coupling coefficient, thus the coupling coefficient is equivalent to the tunneling coefficient. If the photons are inputted into the left waveguide (e.g., waveguide 1) and let $A_2 = 1$, the transmission time in the separation region ($w_1 \leq x \leq w_1 + b$) can be described as (Steinberg 1995)

$$
\tau_T = \frac{m}{\hbar k}\frac{1}{T}\int_{w_1}^{w_1+b} dx\left(A_3e^{k_Bx}+B_3e^{-k_Bx}\right)\left(A_3e^{-k_Bx}+B_3e^{k_Bx}\right) \tag{21}
$$

Note that $w_1 > \lambda$ and $B_3/A_3 \to e^{2k_Bw_1}$, thus $k_Bw_1 \gg 1$ and $B_3 \gg A_3$. This implies that the main contribution to the tunneling time is the B_3^2 term, and thus Eq. (33) leads to

$$
\begin{aligned}
\tau_T &= \frac{mb}{\hbar k}\left|\frac{B_3^2}{A_4}\right| \\
&= \frac{mb}{\hbar k}\times\left|\frac{(ik+k_B)^2\left(e^{2ikw_1}-e^{ikb}\right)}{e^{-2ikw_1}(ik-k_B)^2-(ik+k_B)^2}\right|e^{k_B(4w_1+b)} \\
&\to \frac{m}{\hbar k}e^{\omega\sqrt{\mu\varepsilon}(4w_1+b)}
\end{aligned} \tag{22}
$$

2.6 Uncertainty principle

According to the uncertainty relations between position and momentum and between time and energy, one has

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2} \tag{23}$$

$$\Delta t \cdot \Delta E \geq \frac{\hbar}{2} \tag{24}$$

where Δx is the uncertainty in position, Δp is the uncertainty in momentum, ΔE is the uncertainty in energy, and Δt is the uncertainty in time. Since $E = pc$, one can obtain: $\Delta E = c \cdot \Delta p$ and $\Delta p \geq \hbar/(2c \cdot \Delta t)$.

Since one cannot assure which waveguide a photon is in during the time interval for the photon to travel through the separation layer, thus the uncertainty in the time for the photon can be obtained as $\Delta t = \tau_T$.

The uncertainty principle implies that the more accurate the momentum is decided (i.e., a smaller Δp), the less accurate the position is known (i.e., a larger Δx). Therefore, the uncertainty in the space (the x-direction) can be obtained as

$$\Delta x = \frac{\hbar}{2\Delta p} \leq c \cdot \tau_T \quad \propto \quad \frac{mc}{\hbar k} e^{\omega\sqrt{\mu\varepsilon}(4w_1+b)} \tag{25}$$

The above equation illustrates that the uncertainty in the space (the x-direction) for the photon is the maximum length in the waveguide where the photon can be found. The coupling length is defined as the maximum length of the light transmission along the waveguide (Zheng et al. 2015; Moghaddam et al. 2010; Kuchinsky et al. 2002; Li et al. 2010; Wang et al. 2010; Kumar et al. 1985; Mortensen et al. 2003; Uthayakumar et al. 2012; Obayya et al. 2000; Singh 1999; Gorajoobi et al. 2013; Chaudhari and Gautam 2000; Headley et al. 2004; Suzuki et al. 1994; Song et al. 2011; Sha et al. 2015; Henry and Verbeek 1989; Imotijevic et al. 2007; Yu et al. 2014; Xu et al. 2014; Marom et al. 1984; Djordjev et al. 2002; Zhou et al. 2013; Ali et al. 2008; Bird 1990; Shakouri et al. 1998; Sun 2007; Noda et al. 1981), when the light travels from one waveguide to the adjacent waveguide in coupled waveguides. The coupling length is also defined as the distance for which the maximum guided power is transferred from one medium to the other (Zheng et al. 2015; Moghaddam et al. 2010; Kuchinsky et al. 2002; Li et al. 2010; Wang et al. 2010; Kumar et al. 1985; Mortensen et al. 2003; Uthayakumar et al. 2012; Obayya et al. 2000; Singh 1999; Gorajoobi et al. 2013; Chaudhari and Gautam 2000; Headley et al. 2004; Suzuki et al. 1994; Song et al. 2011; Sha et al. 2015; Henry and Verbeek 1989; Imotijevic et al. 2007; Yu et al. 2014; Xu et al. 2014; Marom et al. 1984; Djordjev et al. 2002; Zhou et al. 2013; Ali et al. 2008; Bird 1990; Shakouri et al. 1998; Sun 2007; Noda et al. 1981). Therefore, the maximum length that the photon can propagate or the uncertainty in the space (the x-direction) for the photon is actually the coupling length, that is, $L_C = c \cdot \tau_T$.

2.7 Coupling length and coupling coefficient

The equations of the coupling length and the coupling coefficient obtained by using the particle properties of light and the traditional methods are compared in below. According to Eqs. (32) and (37), the coupling length and the coupling coefficient based on the uncertainty principle and the tunneling theory can be obtained as

$$L_C = \frac{mc}{\hbar k}\left|\frac{(ik+k_B)^2\left(e^{2ikw_1}-e^{ikb}\right)}{e^{-2ikw_1}(ik-k_B)^2-(ik+k_B)^2}\right|e^{k_B(4w_1+b)} \rightarrow \frac{mc}{\hbar k}e^{\omega\sqrt{\mu\varepsilon}(4w_1+b)} \tag{26}$$

$$P \propto \left|\frac{4k_B^2}{\left(e^{2ikw_1}-e^{ikb}\right)\left[(ik-k_B)^2-e^{2ikw_2}(ik+k_B)^2\right]}\right|e^{-2k_B(2w_1+b)} \rightarrow e^{-2\omega\sqrt{\mu\varepsilon}(2w_1+b)} \tag{27}$$

The coupling length using the modal approach is given as Yariv (1973)

$$L_C = \frac{\pi}{2\kappa} \tag{28}$$

where the parameter κ can be calculated by assuming that the propagation constant of each eigenvalue is essentially the same as that of an unperturbed waveguide via the modal approach and the coupled-mode theory. The parameter κ can be approximated only if the modes are rigorously evaluated. The parameter κ satisfies (Marom et al. 1984; Yariv 1973)

$$\begin{aligned}(-i\beta_a+i\gamma)E_1+\kappa E_2=0\\ -\kappa^*E_1+(-i\beta_b+i\gamma)E_2=0\end{aligned} \tag{29}$$

where $\gamma_{1,2}=\left[\beta_a+\beta_b\pm\sqrt{4\kappa^2+(\beta_a-\beta_b)^2}\right]\Big/2$, and β_a and β_b are the propagating constants of two waveguides, respectively. The above equations have solutions only for the case of identical waveguides (i.e., $\beta_a=\beta_b$). The power ratio between the normal modes (which is herein named as the coupling coefficient) is given as Marom et al. (1984)

$$P = \frac{1+\frac{4\kappa^2}{\left(\sqrt{4\kappa^2+(\beta_a-\beta_b)^2}+(\beta_b-\beta_a)\right)^2}}{1+\frac{4K^2}{\left(\sqrt{4\kappa^2+(\beta_a-\beta_b)^2}-(\beta_b-\beta_a)\right)^2}} \tag{30}$$

For optical directional couplers based on photonic crystal, the coupling length can be determined by Kuchinsky et al. (2002)

$$L_C = 0.5\frac{\Delta\omega}{\omega}V'_{group}\lambda_0 \tag{31}$$

where $\Delta\omega$ is the frequency splitting, V'_{group} is the group velocity in the unit of light velocity in vacuum, and λ_0 is the light wavelength in vacuum. Since there is no analytical expression for the frequency splitting, the coupling length can be determined only by numerical calculations.

Note that the above equations start from the particle properties of light but the standard quantization of the electromagnetic field or to the photon equation of for example Smith and Raymer (2007) start from the wave properties. Thus, we can obtain a new physical solution to the coupling effect based on the particle properties of light. Comparing with the previous models (i.e., Eqs. 40, 43), this new model (i.e., Eqs. 38, 39) has merits of physical simplicity and analytic nature. From this new model, one can readily find how the physical parameters (e.g., the width of the waveguide, the width of the separation layer, and the dielectric constant of the separation layer) affect the coupling length and the coupling coefficient, whereas it is very difficult to directly find the effects of such physical parameters on the coupling length and the coupling coefficient from the previous models. Therefore, this new model is not only beautiful due to its physical simplicity but also has great value in practical applications due to its analytic nature. In the following section, a rich supply of data from nearly 30 published articles will be used to prove the validity of this new model.

3 Results and discussion

Figure 1 depicts how the measured transmission phase time of light changes with the barrier thickness. The experimental data is extracted from Fig. 3 in Balcou and Dutriaux (1997). Herein, in all the figures, symbols denote the data from the literature, where lines represent fitting curves to the data. A good exponential fit for the data of the measured transmission phase time versus the barrier can be clearly observed in Fig. 1. Such an exponential relation between the tunneling time and the barrier thickness is the same as what this new model predicts (i.e., Eq. 23).

Note that there are oscillations in the curve for TE wave. Such an oscillation phenomenon resulted from the quantum interference of material waves has been observed in electrons tunneling through a barrier (Mao et al. 2000a, b, c, 2001a, b, c, 2002; Mao and Wang 2008 and the references therein). Such oscillations for photons may have a similar physical origin as that for electrons because both electron and light exhibit the wave-particle duality. It is also found that such oscillations cannot be easily observed in experiments because of the loss of the quantum interference of material waves (Mao and Wang 2008). Similarly, it can be concluded that those oscillations are difficult to be observed in experiments for light. The values of Adj. R^2 are around 0.99 in the most cases for the data fitting herein.

This new model (i.e., Eq. 38) predicts that the coupling length should exponentially depend on the frequency of light, the width of the waveguide, the width of the separation layer, the square root of the dielectric constant of the separation layer, and the permittivity

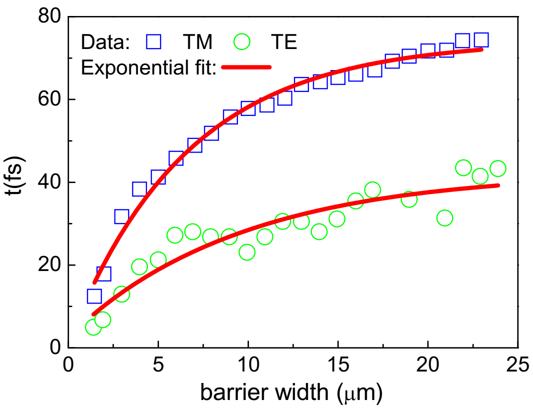


Fig. 1 Impact of the barrier thickness on the transmission phase time. The data is from Fig. 3 in Benderskii and Kats (2002)

of the separation layer. Moreover, this new model (i.e., Eq. 39) also predicts that the coupling coefficient has a similar behavior as the coupling length.

Figure 2 depicts the logarithm of the coupling length versus the width of the separation layer. The data is from Zheng et al. (2015), Li et al. (2010), Singh (1999), Gorajoobi et al. (2013), Chaudhari and Gautam (2000), Suzuki et al. (1994), Song et al. (2011), Sha et al. (2015), Imotijevic et al. (2007), Marom et al. (1984), Yariv (1973). From Fig. 2, it can be clearly observed that the logarithm of the coupling length linearly depends on the width of the separation layer, which is the same as that predicted by this new model (i.e., Eq. 38). The oscillations in the curve of the coupling length versus the width of the separation layer can be observable only in the data from (Chaudhari and Gautam 2000), as shown in panel (d). The physical origin of the oscillations is due to the quantum interference of photons when the particle properties have been considered (Steinberg 1995; Balcou and Dutriaux 1997; Mao et al. 2000a, b, c, 2001a, b, c and the references therein).

Figure 3 depicts the coupling length versus the wavelength of light. The data is from Moghaddam et al. (2010), Kumar et al. (1985), Mortensen et al. (2003), Uthayakumar et al. (2012), Gorajoobi et al. (2013), Henry and Verbeek (1989). From Fig. 3, one can clearly see that there is an exponential relation between the coupling length and the wavelength, which is in good agreement with this new model (i.e., Eq. 38). Red lines in this figure and the following figures represent the fitting curves using one-order exponential functions.

Figure 4 plots the coupling length versus the square root of the relative dielectric constant of the separation layer. The data is from Wang et al. (2010), Obayya et al. (2000), Sun (2007), Noda et al. (1981). One can readily observe that there is an exponential

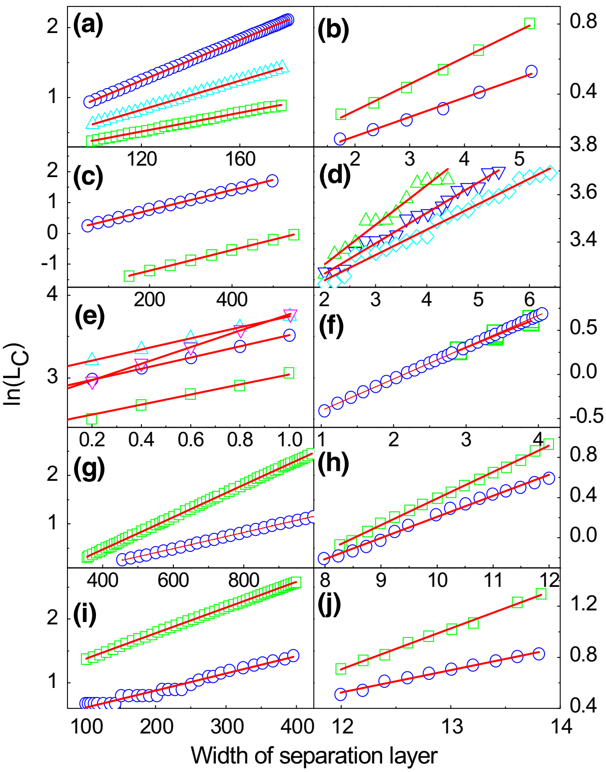


Fig. 2 Impact of the width of the separation layer on the coupling length. **a** Data from Fig. 6 in Zheng et al. (2015), **b** data from Fig. 5 in Singh (1999), **c** square symbols and circle symbols denote data from Fig. 4 in Li et al. (2010) and Fig. 2 in Gorajoobi et al. (2013) respectively, **d** data from Fig. 10 in Chaudhari and Gautam (2000), **e** data from Fig. 2 in Suzuki et al. (1994), **f** data from Fig. 6 in Song et al. (2011), where square and circle symbols denote experimental and simulation data respectively, **g** data from Fig. 2 in Sha et al. (2015), **h** data from Fig. 4 in Imotijevic et al. (2007), **i** data from Fig. 5 in Marom et al. (1984), and **j** data from Fig. 10 in Yariv (1973). The separation is in the unit of μm for (**b**) and (**d**) while in the unit of nm for all the other cases

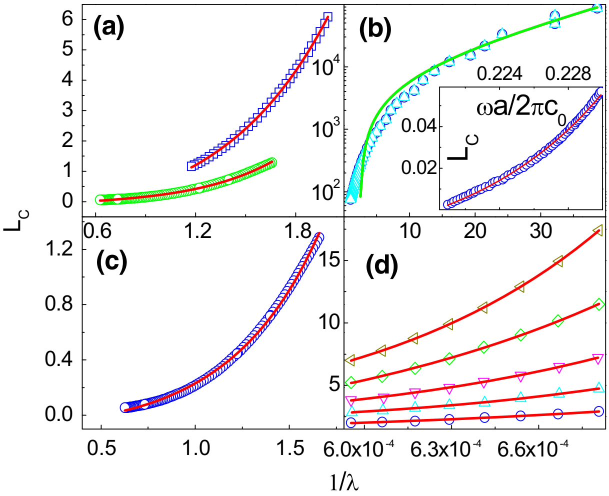


Fig. 3 Impact of the wavelength on the coupling length. **a** Square and circle symbols denote data from Fig. 2 in Kumar et al. (1985) and Fig. 2 in Uthayakumar et al. (2012) respectively, **b** square and circle symbols denote data from Fig. 2 in Mortensen et al. (2003), and the data of the inset from Fig. 2 in Moghaddam et al. (2010) respectively, **c** data from Fig. 2 in Gorajoobi et al. (2013), and **d** data from Fig. 5 in Henry and Verbeek (1989). All Adj. R^2 values are around 0.99

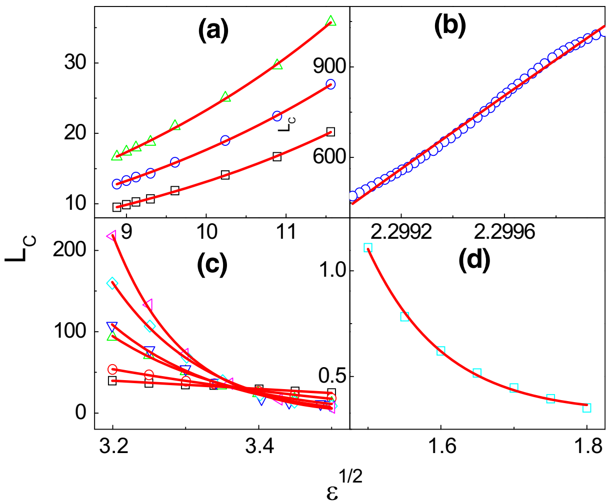


Fig. 4 Impact of the relative dielectric constant of the separation layer on the coupling length. **a** Data from Fig. 2 in Wang et al. (2010), **b** data from Fig. 9 in Obayya et al. (2000), **c** data from Fig. 3 in Sun (2007), and **d** data from Fig. 4 in Noda et al. (1981)

relation between the coupling length and the square root of the relative dielectric constant of the separation layer, which is again in good agreement with this new model (i.e., Eq. 38). Small oscillations in the curve of the coupling length versus induced index difference can be observable only in the data from Obayya et al. (2000). All these oscillations may have the same physical origin as that for electrons because the physics of tunneling for classical light waves is the same as that for quantum mechanical waves (Steinberg 1995, and the references therein).

Figure 5 depicts how the coupling length changes with the width of the waveguide. The data is from Yu et al. (2014), Xu et al. (2014). From Fig. 5, one can clearly see that there is an exponential relation between the coupling length and the width of the waveguide, which is in good agreement with this new model (i.e., Eq. 38). Figures 2, 3, 4 and 5 validate this new model, which can well predict how the width of the separation layer, the wavelength of light, the square root of the relative dielectric constant of the separation layer, and the width of the waveguide affect the coupling length.

Figure 6 shows the coupling coefficient as a function of the width of the separation layer. The data is from Li et al. (2010), Djordjev et al. (2002), Zhou et al. (2013). Ali et al. (2008), Bird (1990). From Fig. 6, an exponential relation between the coupling coefficient and the width of the separation layer can be observed, which is exactly the same as that predicted by this new model (i.e., Eq. 39). Small oscillations in the curve of the coupling coefficient versus the width of the separation layer can be observable only in the data from Ali et al. (2008).

Figure 7 plots the coupling coefficient versus the frequency of light. The data is from Moghaddam et al. (2010), Shakouri et al. (1998). From Fig. 7, it can be clearly observed that there is an exponential relation between the coupling coefficient and the frequency (wavelength) of light, which is again exactly as the same as that predicted by this new model (i.e., Eq. 39). Obvious oscillations in the curve of the coupling coefficient versus the frequency can be observable only in the data from Moghaddam et al. (2010). The reason for such oscillations has been discussed in the preceding. Figures 6 and 7 validate this new model, which can exactly predict how the width of the separation layer and the frequency of light affect the coupling coefficient.

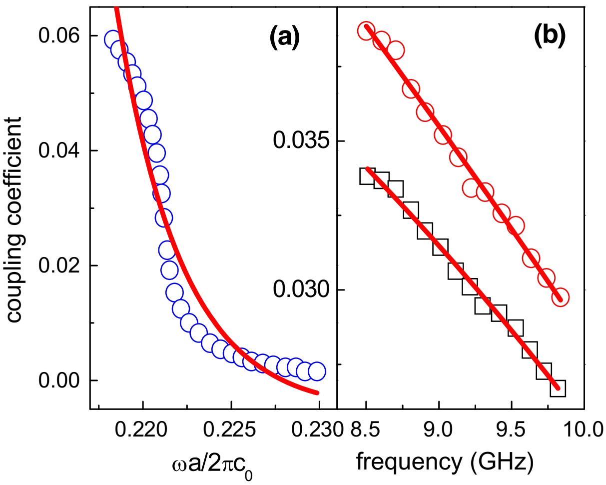


Fig. 5 Impact of the waveguide width on the coupling length. **a** Data from Fig. 5 in Yu et al. (2014), and **b** data from Fig. 2 in Xu et al. (2014)

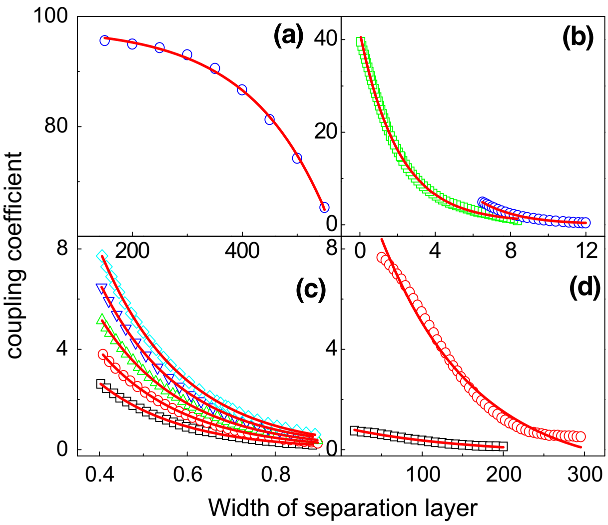
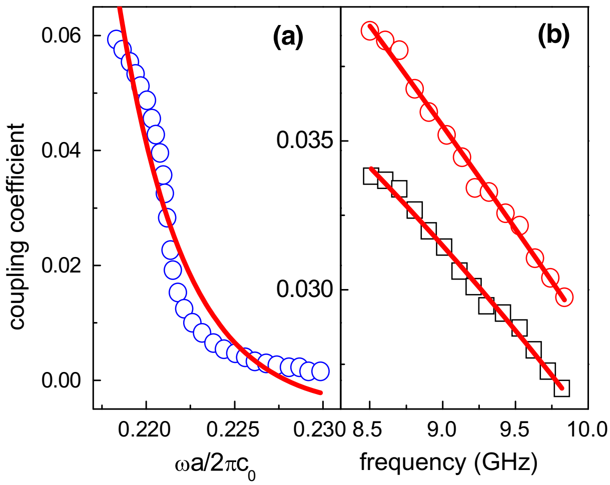


Fig. 6 Impact of the width of the separation layer on the coupling coefficient. **a** Data from Fig. 4 in Li et al. (2010), **b** square and circle symbols denote Fig. 5 in Djordjev et al. (2002) and Fig. 7 in Bird (1990), **c** data from Fig. 7 in Zhou et al. (2013), and **d** data from Fig. 3 in Ali et al. (2008)

Fig. 7 Impact of the frequency and wavelength on the coupling coefficient. **a** Data from Fig. 2 in Moghaddam et al. (2010), and **b** data from Fig. 3 in Shakouri et al. (1998)



4 Conclusions

A method for evaluating the coupling length and the coupling coefficient of parallel coupled channel waveguides was proposed. This new model is based on the wave-particle duality of light by virtue of the newly introduced concepts of the virtual mass and potential for a photon, the uncertainty principle and the tunneling theory in quantum mechanics, which is different from the previous method that uses a close mathematical analogy between the Schrödinger equation and the Helmholtz equation to solve the optic problem or the electromagnetic problem.

From the particle properties of light (i.e., the light behaves both as a particle and as a wave), the Schrödinger equation for a photon was firstly established via the assumption that the largest photon momentum corresponds the fastest speed or the lowest potential, and thus the potential in the vacuum is set as zero. Secondly, the law of the coupling coefficient was deduced by solving the Schrödinger equation of a photon with assistance of the particle tunneling theory in quantum mechanics. The particle properties of light show that a photon through a dielectric layer behaves like a hole (when $\varepsilon\mu \geq \varepsilon_0\mu_0$) or an electron (when $\varepsilon\mu \leq \varepsilon_0\mu_0$) through a barrier. Then, considering that the coupling length is caused by the uncertainty in the space for the photon, the law of the coupling length was also deduced by virtue of the uncertainty principle and the tunneling theory.

By analyzing the experimental (or simulation) data in nearly 30 previously published articles, one can observe that the coupling length and the coupling coefficient are exponentially dependent on the frequency of light, the width of the waveguide, the width of the separation layer, and the square root of the dielectric constant of the separation layer, which can be well predicted by this new model.

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Appendix

C Hole-like carrier tunneling: the wave functions in three regions: $\psi(x) = e^{ikx} + \mathrm{Re}^{-ikx}$ $(x < 0)$; $\psi(x) = Ae^{-k_Bx} + Be^{k_Bx}(0 \leq x \leq)b$; $\psi(x) = Te^{\mathrm{ik}x}(x > b)$. Here $k = \frac{\sqrt{2mE}}{\hbar}$, $k_B = \frac{\sqrt{2m\varphi}}{\hbar} = \omega\sqrt{\mu\varepsilon}$. Matching the wave functions and their derivatives at the boundaries $(x = 0$ and $x = b)$ leads to:$1 + R = A + B$, $Ae^{-k_Bb} + Be^{k_Bb} = Te^{ikb}$, $-k_BAe^{-k_Bb} + k_BBe^{k_Bb} = ikTe^{ikb}$, and thus

$$T = e^{-ikb}\frac{4ikk_B}{(k_B + ik)^2e^{-k_Bb} - (k_B - ik)^2e^{k_Bb}} \tag{32}$$

$$R == \frac{\left[(k_B)^2 + k^2\right]e^{-k_Bb} - \left[(k_B)^2 + k^2\right]e^{k_Bb}}{(k_B + ik)^2e^{-k_Bb} - (k_B - ik)^2e^{k_Bb}} \tag{33}$$

$$A = \frac{2ik(k_B - ik)e^{k_Bb}}{(k_B + ik)^2e^{-k_Bb} - (k_B - ik)^2e^{k_Bb}} \tag{34}$$

$$B = \frac{2ik(k_B + ik)e^{-k_Bb}}{(k_B + ik)^2e^{-k_Bb} - (k_B - ik)^2e^{k_Bb}} \tag{35}$$

E Photon tunneling in double waveguides: the wave functions in five regions are

$$\text{Region 1 } (x<0): \ \psi(x) = B_1e^{k_Bx}$$

$$\text{Region 2 } (0 \leq x \leq w_1): \psi(x) = A_2e^{ikx} + B_2e^{-ikx}$$

$$\text{Region 3 } (w_1 \leq x \leq w_1 + b): \ \psi(x) = A_3e^{-k_Bx} + B_3e^{k_Bx}$$

$$\text{Region 4 } (w_1 + b < x < w_1 + w_2 + b): \; \psi(x) = A_4 e^{-k_B x} + B_4 e^{k_B x}$$

$$\text{Region 5 } (x > w_1 + w_2 + b): \; \psi(x) = A_5 e^{-k_B x}$$

Matching the wave functions and their derivatives at the boundaries, one can obtain the following equations:

$$\frac{B_1}{A_2} = \frac{ik}{ik + k_B} \tag{36}$$

$$\frac{B_2}{A_2} = \frac{ik - k_B}{ik + k_B} \tag{37}$$

$$\frac{A_3}{A_2} = \frac{2k_B(ik + k_B)e^{-k_B w_1}}{(k_B^2 + k^2)(e^{ikw_1} - e^{-ikw_1})} \tag{38}$$

$$\frac{B_3}{A_2} = \frac{2k_B(ik + k_B)e^{k_B w_1}}{e^{ikw_1}(ik + k_B)^2 - e^{-ikw_1}(ik - k_B)^2} \tag{39}$$

$$\frac{A_4}{A_3} = \frac{2k_B(ik - k_B)e^{-k_B(w_1 + b)}}{e^{ik(w_1+b)}\left[e^{2ikw_2}(ik + k_B)^2 - (ik - k_B)^2\right]} \tag{40}$$

$$\frac{B_4}{A_4} = \frac{(ik + k_B)}{(ik - k_B)} e^{2ik(w_1 + w_2 + b)} \tag{41}$$

$$\frac{A_4}{A_5} = \frac{e^{-k_B(w_1 + w_2 + b)}}{e^{ik(w_1 + w_2 + b)} + \frac{(ik + k_B)}{(ik - k_B)} e^{ik(w_1 + w_2 + b)}} \tag{42}$$

$$T = \frac{A_4}{A_2} = \frac{4k_B^2 e^{-k_B(2w_1 + b)}}{(e^{2ikw_1} - e^{ikb})\left[(ik - k_B)^2 - e^{2ikw_2}(ik + k_B)^2\right]} \tag{43}$$

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